Quarks and leptons

- Peskin 20.3
- Cheng-Li 11.3
- Schwartz 29

The SM contains a number of spin-1/2 fermionic fields, collectively denoted as "matter fermions".

Only a few of them are actual constituents of ordinary matter, the majority needs to be produced at colliders.

There are two types of matter fermions:

- quarks (charged under SU(3))
- leptons (neutral under SU(3))

It is a fact of life that they both contain three copies with identical quantum numbers.

- Leptons

  One family of leptons consists of

3 Weyl spinors, represented by Dirac spinors $\psi_{L,R}$ with

$$\gamma^5 \psi_L = - \psi_L \quad , \quad \gamma^5 \psi_R = + \psi_R$$

The lepton fields are

$$\ell_L \, , \, \ell_R \, , \, \nu_L$$

It is possible that a right-handed neutrino ν_R also exists, but we don't know. Therefore we do not include it in our SM definition.

Focusing on the electroweak part of the SM, so $SU(2)_L \times U(1)_Y$, we should specify how the fields transform.

The left-handed fields form a SU(2) doublet,

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$$

which transf. under fundamental of SU(2).

We want to identify ℓ_L with the electron, so we should chose the hyperchage so that $Q(l_L) = -1$. Using $Q = T^3 + Y$,

we need $Y(l_L) = -1/2$.

Note that this implies $Q(\nu_L) = 0$, so the neutrinos are neutral.

We are obligued to assign the same hypercharge to ν_L and l_L because they are part of the same doublet, and Y must commute with $SU(2)_L$ generators.

The right-handed electron l_R is instead a singlet of $SU(2)_L$, so

$$Y(l_R) = -1.$$

If ν_R exists, it must be a total singlet of the group, $T^a(\nu_R) = Y(\nu_R) = 0$. This is why we are unsure of its existence: it does not participate in weak interactions and it is extremelly weakly coupled to other interactions.

In summary,

$$L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \in \underset{SU(2)}{2}{}_{\underset{U(1)_Y}{1/2}} \qquad \ell_R \in 1_{-1}$$

- Quarks

One family of quarks is made of

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in 2_{1/6} \quad , \quad u_R \in 1_{2/3} \quad , \quad d_R \in 1_{-1/3}$$

The fractional charges are determined by the requirement that they are constituents of the proton & neutron.

The quarks have an additional quantum number, the "color". Each u & d field have an extra index $i=1,2,3$, so that they are in the fundamental of the gauge group $SU(3)$. We will discuss this when introducing QCD interactions.

- Families

Both quarks & leptons come in 3 copies, or families:

I	II	III
ν_e, e	ν_μ, μ	ν_τ, τ
u, d	c, s	t, b

Families are identical from the viewpoint of gauge interactions.

- By the minimal gauging prescription we can work out the matter couplings to $W^\pm, Z, A$.

Recall that

$$W_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}\, W_\mu^+ \\ \sqrt{2}\, W_\mu^- & -W_\mu^3 \end{pmatrix}$$

For a doublet ψ_L,

$$D_\mu \psi_L = (\partial_\mu \mathbb{1} - i g W_\mu - i g' Y(\psi_L) B_\mu \mathbb{1})\, \psi_L$$

and for a singlet ψ_R

$$D_\mu \psi_R = (\partial_\mu - i g' Y(\psi_R) B_\mu)\, \psi_R$$

This gives "charged current" interactions for

$$\psi_L = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

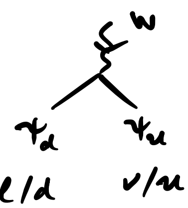
$$\mathcal{L} \supset \frac{1}{\sqrt{2}} g\, \bar{\psi}_{u,L} \gamma^\mu W^+_\mu \psi_{d,L} + \frac{1}{\sqrt{2}} g\, \bar{\psi}_{d,L} \gamma^\mu W^-_\mu \psi_{u,L}$$

If we want to express this in terms of

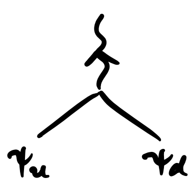
$\psi = \psi_L + \psi_R$,

$$\mathcal{L} \supset \frac{g}{2\sqrt{2}} \bar{\psi}_u \gamma^\mu (1-\gamma^5) W^+_\mu \psi_d$$

$$+ \frac{g}{2\sqrt{2}} \bar{\psi}_d \gamma^\mu (1-\gamma^5) W^-_\mu \psi_u$$

$\rightarrow$ 

$$= i \frac{g}{2\sqrt{2}} \gamma^\mu (1-\gamma^5)$$



$$= i \frac{g}{2\sqrt{2}} \gamma^\mu (1-\gamma^5)$$

Note the universality of the couplings.

We can write it in terms of a "charged current"

$$J_\mu^+ = \sum_{\text{doubl.}} \bar{\psi}_d \gamma_\mu (1-\gamma^5) \psi_u$$

$$= \bar{e} \gamma_\mu (1-\gamma^5) \nu + \bar{d} \gamma_\mu (1-\gamma^5) u \quad + \text{other families}$$

and

$$J_\mu^- = (J_\mu^+)^\dagger$$

$$\Rightarrow \quad \mathcal{L} \supset \frac{g}{2\sqrt{2}} W_\mu^+ J^{-\mu} + \frac{g}{2\sqrt{2}} W_\mu^- J^{+\mu}$$

- Neutral currents follow the same logic

$$\mathcal{L} \supset \sum_{i=u,d} \bar{\psi}_{i,L} \gamma^\mu \left(g T^3(i) W_\mu^3 + g' Y B_\mu \right) \psi_{i,L}$$

Using that $Q = T^3 + Y$,

$$= \sum_i \bar{\psi}_{i,L} \gamma^\mu \Big((g W_\mu^3 - g' B_\mu) T^3(i) + g' Q(i) B_\mu \Big) \psi_{i,L}$$

Remembering that

$$W^3 = c_w Z + s_w A$$

$$B = c_w A - s_w Z$$

We find the photon interactions

$$\mathcal{L}^{(A)} = \sum_i e\, Q(i)\, \bar{\Psi}_{i,L} \gamma^\mu A_\mu \Psi_{i,L} = \mathcal{L}_{QED}$$

We recover QED as it is the unbroken symm.

For the Z,

$$\mathcal{L}^{(Z)} = \frac{g}{c_W} \sum_i \bar{\Psi}_{i,L} \gamma^\mu Z_\mu \left[T^3(i) - s_W^2 Q(i) \right] \Psi_{i,L}$$

- For the right-handed fields,

$$\mathcal{L} = g' B_\mu Y(\Psi_R) \bar{\Psi}_R \gamma^\mu \Psi_R$$

$$= e\, Q(\Psi) \bar{\Psi}_R \not{A} \Psi_R - \frac{g}{c_W} s_W^2 Q(\Psi) \bar{\Psi}_R \not{Z} \Psi_R .$$

For the photon we recover QED & for the Z it couples to neutral current

$$\mathcal{L}^{N.C.} = \frac{g}{c_W} Z_\mu J^{0\mu}$$

$$J^{0\mu} = \sum_{\text{all } \Psi\text{'s}} \bar{\Psi}_i \gamma^\mu \left(g_V(i) - g_A(i) \gamma^5 \right) \Psi_i$$

with axial & vector couplings

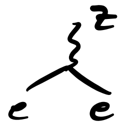
$$g_V^{(i)} = \frac{1}{4}\left(2\,T^3(i) - 4 s_w^2\, Q(i)\right)$$

$$g_A^{(i)} = \frac{1}{4}\left(2\,T^3(i)\right)$$

T^3 is left-handed comp. of Dirac field.

- Interactions:

$= i e Q \gamma^\mu$ like in QED



$$= i \frac{g}{4c_w} \gamma^\mu \left((-1 + 4 s_w^2) - \gamma^5 \right)$$

$$= i \frac{g}{4c_w} \gamma^\mu (1 - \gamma^5)$$ ← only the left neutrino interacts

And similar for quarks.
All tree families couple in the same way. This universality of neutral couplings plays an important role in phenomenology of SM.

- At the moment, note that there is a large $U(3)_L \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$ global symmetry. We will see how Yukawa interactions break it down to $U(1)_B \times U(1)_L$

- The fact that $SU(2)_L$ couples to left-handed fermions makes the discussion of P & C symmetries nontrivial.

- Parity

For the photon,

$$A_\mu(x) \xrightarrow{P} P_\mu{}^\nu A_\nu(x^{(P)}) \qquad x^{(P)\mu} = P^\mu{}_\nu x^\nu$$

$$P^\mu{}_\nu = \mathrm{diag}(+---)$$

and for arb. gauge theory,

$$A_\mu^a \longrightarrow P_\mu{}^\nu A_\nu^a(x^P)$$

So

$$F_{\mu\nu}^a \longrightarrow P_\mu{}^\lambda P_\nu{}^\rho F_{\lambda\rho}(x^P)$$

And the standard action

$$S = \int d^4x \, \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} \rightarrow \int d^4x \, \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}(x)$$

is P invariant. Note that

$$\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \xrightarrow{P} -\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

but in perturbation theory this term has no effect.

The Higgs is a scalar, $H \xrightarrow{P} H$, so the entire bosonic sector preserves P.

- on Dirac fermion fields,

$$\psi(x) \xrightarrow{P} \gamma^0 \psi(x^P) \qquad \text{(e.g. P-S 3.6)}$$

So the vector current

$$\bar{\psi}_1 \gamma^\mu \psi_2 \xrightarrow{P} \bar{\psi}_1 \gamma^0 \gamma^\mu \gamma^0 \psi_2 = P^\mu{}_\nu \, \bar{\psi}_1 \gamma^\nu \psi_2$$

and the axial current

$$\bar{\psi}_1 \gamma^\mu \gamma^5 \psi_2 \rightarrow \bar{\psi}_1 \gamma^0 \gamma^\mu \gamma^5 \gamma^0 \psi_2 = -P^\mu{}_\nu \, \bar{\psi}_1 \gamma^\nu \psi_2$$

P is a symmetry of QED because the photon couples to the vector current.

This is because both L & R fields have the same charges under $U(1)_{em}$.

Parity is instead maximally broken by EW interactions

$$J_\mu^+ = \sum_i \bar{\psi}_i \gamma_\mu (1-\gamma^5) \psi_i$$

$$J_\mu^0 = \sum_i \bar{\psi}_i \gamma_\mu (g_V^i - g_A^i \gamma^5) \psi_i$$

So P is not a symmetry of the SM.

The SM is a chiral gauge theory.

- Charge conjugation

Charge conjugation is a bit more involved because we need to understand what it means to conjugate a non-abelian charge.

What we want is

$$T^A \to -(T^A)^* = -(T^A)^T$$

Commutation relations are preserved under CC:

$$[T^A, T^B] = i f^{ABC} T^C$$

$$\hookrightarrow \quad [T^{A*}, T^{B*}] = -i f^{ABC} T^{C*}$$

$$\hookrightarrow \quad [-T^{A*}, -T^{B*}] = i f^{ABC} (-T^C)^*$$

Complex conjugation can be shown to be always an automorphism.

This means, $\exists\ c_{AB}$ s.t. $-T^{A*} = c^{AB} T^B$

The proof (Georgi, ch 28) follows from the fact that generators can be either pure imaginary or real. Then

$$c^{AB} = \begin{pmatrix} -\mathbb{1}_{real} & 0 \\ 0 & \mathbb{1}_{im} \end{pmatrix}$$

For instance, for $SU(2)$ we have

$$\begin{aligned} W_\mu^{1,3} &\xrightarrow{c} -W_\mu^{1,3} \\ W_\mu^2 &\longrightarrow +W_\mu^2 \end{aligned} \quad \Rightarrow \quad \left\{ \begin{aligned} W_\mu^{\pm} &\longrightarrow -W_\mu^{\mp} \\ W_\mu^3 &\longrightarrow -W_\mu^3 \end{aligned} \right.$$

In the abelian case,

$$B \to -B \quad \Rightarrow \quad \begin{array}{l} Z \xrightarrow{C} -Z \\ A \xrightarrow{C} -A \end{array}$$

The kinetic term is always invariant

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu] \to -F_{\mu\nu}^* = -F_{\mu\nu}^T$$

$$\Rightarrow \quad tr(F^2) \xrightarrow{C} tr(F^2)$$

- For the Higgs, $H(x) \xrightarrow{C} H^*(x)$

$$D_\mu H = \partial_\mu H - i W_\mu H - \frac{i}{2} B_\mu H$$

$$\hookrightarrow \quad C: \to \partial_\mu H^* + i W_\mu^* H^* + \frac{i}{2} B H^* = (D_\mu H)^*$$

So the entire bosonic sector is C invariant.
It is also P-inv, so it is CP-invariant.

- If we keep all components of the Higgs doublet,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\sqrt{2}\,\pi_+ \\ v + h + i\pi_0 \end{pmatrix}$$

So under C: $\pi_+ \to -\pi_- = -\pi_+^*$, $\pi_0 \to -\pi_0$

$h \to h$

So physical higgs is CP even and Goldstones are CP odd and transf. as they need to be consistent with being long. modes of $W^\pm / Z$.

- Fermions.

For Dirac fermions,

$$\psi \xrightarrow{C} C\bar{\psi}^T \qquad C = i\gamma^2\gamma^0$$

$$\bar{\psi} \xrightarrow{C} \psi^T C \qquad \downarrow$$

$$C^* = C, \quad C^2 = -\mathbb{1}$$

using $C\gamma^\mu C = (\gamma^\mu)^T$, $[C, \gamma^5] = 0$,

$$\bar{\psi}_i \gamma^\mu \psi_j \xrightarrow{C} -\bar{\psi}_j \gamma^\mu \psi_i$$

$$\bar{\psi}_i \gamma^\mu \gamma^5 \psi_j \xrightarrow{C} +\bar{\psi}_j \gamma^\mu \gamma^5 \psi_i$$

Therefore, using

$$\mathcal{L}_{cc} = W_\mu^+ J^{\mu -} + h.c.$$

$$\mathcal{L}_{nc} = Z_\mu J^{0\mu}$$

the vector coupling preserves charge but the axial doesn't.

So SM gauge interactions, being chiral, break C.

But, while they break both P & C, the combined CP transformation is preserved.

For now...